

QUARKONIUM PRODUCTION AT Z^0 AND IN $\Upsilon(1S)$ DECAY

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The conventional color-singlet model was challenged by the recent data on quarkonium production. Discrepancies in production rates were observed at the Tevatron, at LEP, and in fixed-target experiments. The newly advocated color-octet mechanism provides a plausible solution to the anomalous quarkonium production observed at the Tevatron. The color-octet mechanism should also affect other quarkonium production channels. In this paper we will summarize the studies of quarkonium production in Z^0 and Υ decays.

1. Introduction

Charmonium was first discovered more than twenty years ago, which was believed to be a pair of charm and anti-charm quarks bounded by strong QCD forces. The production and decay of quarkonia had then become very interesting subjects in QCD. The production of quarkonia was first formulated by the color evaporation model¹. It states that the production rate of a particular quarkonium state H is a certain fraction f_H of the open charm pair production with invariant mass $m_{c\bar{c}}$ between $2m_c$ and $2m_D$, i.e., for example,

$$\sigma(\psi) = f_\psi \times \int_{2m_c}^{2m_D} \frac{\sigma(c\bar{c})}{dm_{c\bar{c}}} dm_{c\bar{c}} . \quad (1)$$

The model has the predictive power that f_H 's are process independent, and can reproduce the \sqrt{s} dependence of the total cross section. However, it is unable to predict the relative rates for different spin-orbital states.

The color evaporation model was later superseded by the color-singlet model (CSM)². This model is based on the assumption that the spin-orbital angular momentum and the color of the asymptotic physical state is the same as the point-like $c\bar{c}$ pair produced in the short distance region. Thus, the amplitude involving

a physical charmonium state is proportional to the amplitude involving the point-like $c\bar{c}$ pair in the same angular momentum state with $c\bar{c}$ in a color-singlet state and having small relative momentum. The proportionality constant is given by the wavefunctions or derivatives of wavefunctions evaluated at the origin, which are obtained by solving the Schrödinger equation of the c and \bar{c} system.

The CSM is simple with high predictive power. However, it often gives quarkonium production rates below those measured from the fixed-target experiments. The contrast becomes much prominent when measured rates of prompt ψ and ψ' in CDF experiment³ exceed those predicted by the CSM by orders of magnitude at the large transverse momentum region. The inadequacy of CSM led to two important theoretical developments in quarkonium production⁴. The first one is the idea of gluon⁵ and heavy quark fragmentation⁶. Though these fragmentation processes are higher order corrections in α_s , they have been shown⁷ to be more important than the lowest order QCD production⁸ in the large p_T region. The dominant contribution to ψ production would be gluon fragmentation $g^* \rightarrow \chi_{cJ}$ followed by the radiative decays $\chi_{cJ} \rightarrow \psi\gamma$. For ψ' the dominant contributions come from gluon and charm quark fragmentation. However, the ψ data showed that the ψ fed down from χ_{cJ} decay is not the major source of ψ^3 . Moreover, the ψ' data was still orders of magnitude above the prediction even including fragmentation contributions.

The second important development is the color-octet mechanism⁹, which includes higher order corrections in v^2 , where v is the relative velocity of c and \bar{c} inside the charmonium. Within Non-Relativistic QCD (NRQCD)¹⁰ formalism of the quarkonium, the physical state, e.g. ψ , is not solely a $c\bar{c}$ pair in 3S_1 color-singlet state, instead, a superposition of all Fock states:

$$\begin{aligned} |\psi\rangle &= |c\bar{c}(^3S_1, \underline{1})\rangle + \mathcal{O}(v) |c\bar{c}(^3P_J, \underline{8})g\rangle + \mathcal{O}(v^2) |c\bar{c}(^3S_1, \underline{8} \text{ or } \underline{1})gg\rangle \\ &+ \mathcal{O}(v^2) |c\bar{c}(^1S_0, \underline{8})g\rangle + \mathcal{O}(v^2) |c\bar{c}(^3D_{J'}, \underline{8} \text{ or } \underline{1})gg\rangle + \mathcal{O}(v^3), \end{aligned} \quad (2)$$

where the angular momenta of the $c\bar{c}$ pair in each Fock state is labeled by $^{2S+1}L_J$ with a color configuration of either $\underline{8}$ or $\underline{1}$. Similar expressions exist for χ_{cJ} states. Production of ψ can now be via higher Fock states with emission or absorption of dynamical gluons. The NRQCD factorization formula for the production of a quarkonium state H is given by

$$d\sigma(H + X) = \sum_n d\hat{\sigma}(c\bar{c}(n) + X) \langle \mathcal{O}_n^H \rangle, \quad (3)$$

where $d\hat{\sigma}$ is the short distance factor producing the $c\bar{c}$ pair in an intermediate state n (n is a collective notation for angular momentum and color), and $\langle \mathcal{O}_n^H \rangle$ is the matrix element describing the transition of the state n into the physical state H . The relative importance of each term in (3) depends on the order of α_s in $d\hat{\sigma}$ and v^2 in $\langle \mathcal{O}_n^H \rangle$. The CSM is essentially the leading term in the expansion (3). The best known example^{9,11,12} of color-octet mechanism is the gluon fragmentation into a $c\bar{c}(^3S_1, \underline{8})$ intermediate state, which then evolves nonperturbatively into the physical ψ or ψ' . This is so far the best plausible solution to the anomalous ψ and ψ' production at the large p_T region in hadronic collisions.

If the color-octet mechanism is correct, it would have significant impact on other production modes, e.g., photoproduction¹³, hadroproduction¹⁴, production in $Z^{015,16}$, Υ^{17} , and B decays¹⁸, and at e^+e^- collisions¹⁹. While other modes are summarized by other speakers, we concentrate on the production in Z^{015} and Υ^{17} decays.

2. ψ Production in Z^0 Decay¹⁵

Though the major source of ψ in Z^0 decay comes from B meson decays, the LEP detectors are able to separate the prompt ψ from the ψ coming from B decay. The recent data on prompt ψ production showed a factor of almost 10 larger than the prediction by the CSM. Therefore, it would be interesting to examine the effects of the color-octet mechanism in this production mode.

Let us first summarize a few important color-singlet processes. The leading order color-singlet processes are $Z \rightarrow \psi gg^{20}$ and $Z \rightarrow \psi c\bar{c}^{21}$. Although both processes are of order α_s^2 , the latter is two orders of magnitude larger. The latter process has been interpreted as a fragmentation process⁶ in which the $c\bar{c}$ pair was first produced almost on-shell from the Z decay and then followed by the fragmentation of c or \bar{c} into the ψ^6 . Therefore, the process $Z \rightarrow \psi c\bar{c}$ is not suppressed by the quark propagator effect of order $1/M_Z$ as it does in the process $Z \rightarrow \psi gg$. In the fragmentation approximation, the energy distribution of ψ in the process $Z \rightarrow \psi c\bar{c}$ is given by⁶

$$\frac{d\Gamma}{dz} (Z \rightarrow \psi(z) c\bar{c}) \approx 2 \Gamma(Z \rightarrow c\bar{c}) \times D_{c \rightarrow \psi}(z), \quad (4)$$

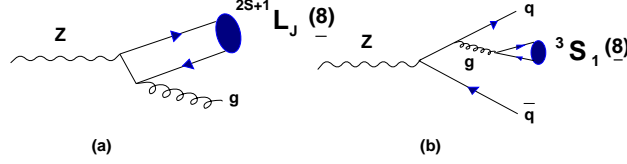
where

$$D_{c \rightarrow \psi}(z) = \frac{16\alpha_s^2(2m_c)\langle\mathcal{O}_1^\psi(^3S_1)\rangle}{243m_c^3} \frac{z(1-z)^2}{(2-z)^6} (16 - 32z + 72z^2 - 32z^3 + 5z^4), \quad (5)$$

and $z = 2E_\psi/M_Z$ with E_ψ denotes the energy of ψ . Both processes involve the same matrix element $\langle\mathcal{O}_1^\psi(^3S_1)\rangle$ whose value can be extracted from the leptonic width to be about 0.73 GeV^3 . Numerically, the widths for $Z \rightarrow \psi gg$ and $Z \rightarrow \psi c\bar{c}$ are about $6 \times 10^{-7} \text{ GeV}$ and $7 \times 10^{-5} \text{ GeV}$, respectively. There is also a higher order color-singlet process $Z \rightarrow q\bar{q}g^*$ followed by the gluon fragmentation²² $g^* \rightarrow \psi gg$. The branching ratio was estimated to be of order 10^{-6} only^{5,22}. However, the recent results from LEP²³ showed a branching ratio of order 10^{-4} for prompt ψ production, which is well above all the predictions from the color-singlet model.

The leading order color-octet process is of order α_s given by the process $Z \rightarrow \psi g$, for which one of the Feynman diagrams is shown in Fig. 1(a). Unfortunately, the short-distance factors are suppressed by at least one power of $1/M_Z$. Numerically, the width for $Z \rightarrow \psi g$ is of order 10^{-7} GeV which renders this process useless at the Z resonance.

The dominant color-octet process actually begins at the order α_s^2 in the process $Z \rightarrow \psi q\bar{q}$, for which one of the Feynman diagrams is shown in Fig. 1(b). Here q represents u, d, s, c , or b . The energy distribution of ψ for the process $Z \rightarrow \psi q\bar{q}$ is

Fig. 1. Representative Feynman diagrams for (a) $Z \rightarrow \psi g$, and (b) $Z \rightarrow \psi q \bar{q}$.

calculated, in the limit $m_q = 0$, to be¹⁵

$$\begin{aligned} \frac{d\Gamma}{dz}(Z \rightarrow \psi(z)q\bar{q}) &= \frac{\alpha_s^2(2m_c)}{18} \Gamma(Z \rightarrow q\bar{q}) \frac{\langle O_8^\psi(^3S_1) \rangle}{m_c^3} \\ &\times \left[\left(\frac{(z-1)^2 + 1}{z} + 2\xi \frac{2-z}{z} + \xi^2 \frac{2}{z} \right) \log \left(\frac{z+z_L}{z-z_L} \right) - 2z_L \right], \end{aligned} \quad (6)$$

where $z = 2E_\psi/M_Z$, $\xi = M_\psi^2/M_Z^2$, and $z_L = (z^2 - 4\xi)^{1/2}$. The physical range of z is $2\sqrt{\xi} < z < 1 + \xi$. Integrating Eq. (6) over the physical range of z gives the decay width of $Z \rightarrow \psi q \bar{q}$ ¹⁵

$$\begin{aligned} \frac{\Gamma(Z \rightarrow \psi q \bar{q})}{\Gamma(Z \rightarrow q \bar{q})} &= \frac{\alpha_s^2(2m_c)}{36} \frac{\langle O_8^\psi(^3S_1) \rangle}{m_c^3} \left\{ 5(1 - \xi^2) - 2\xi \log \xi + (1 + \xi)^2 \times \right. \\ &\left. \left[2\text{Li}_2\left(\frac{\xi}{1+\xi}\right) - 2\text{Li}_2\left(\frac{1}{1+\xi}\right) - 2\log(1+\xi)\log\xi + 3\log\xi + \log^2\xi \right] \right\}, \end{aligned} \quad (7)$$

where $\text{Li}_2(x) = -\int_0^x \frac{dt}{t} \log(1-t)$ is the Spence function. Numerically, the above ratio is 2.2×10^{-4} . Summing over all the quark flavors ($q = u, d, s, c, b$), we obtain the decay width $\sum_q \Gamma(Z \rightarrow \psi q \bar{q}) \simeq 3.8 \times 10^{-4}$ GeV and the branching ratio $\sum_q \text{Br}(Z \rightarrow \psi q \bar{q}) \simeq 1.5 \times 10^{-4}$. Assuming the dominant prompt ψ production process to be $Z \rightarrow q \bar{q} g^*$ with $g^* \rightarrow \psi + X$, in which according to color-singlet model the off-shell gluon fragments into a ψ plus two perturbative gluons, DELPHI²³ obtained the limit $\text{Br}(Z \rightarrow q \bar{q} g^*; g^* \rightarrow \psi + X) < 4.1 \times 10^{-4}$. Since the event topology of our color-octet process is similar to this one, this limit should also be valid for our color-octet process. Therefore, our result is consistent with this data.

In Fig. 2 we compare the energy distributions of ψ coming from the most important color-octet process $Z \rightarrow \psi q \bar{q}$ using Eq. (6) and the most dominant color-singlet process $Z \rightarrow \psi c \bar{c}$ using Eq. (4). The comparison in Fig. 2 shows a very spectacular difference between the color-octet and color-singlet processes. The spectrum for the color-octet process is very soft with a pronounced peak at the lower z end, while the spectrum for the color-singlet process is rather hard because of the nature of heavy quark fragmentation⁶.

Another possible test to distinguish the charm quark fragmentation $Z \rightarrow \psi c \bar{c}$ and the color-octet process $Z \rightarrow \psi q \bar{q}$ is using the polarization of ψ . The polarization of ψ can be measured by the angular distribution of the lepton pair in the rest frame of ψ . The charm quark fragmentation process predicts a polarization ratio

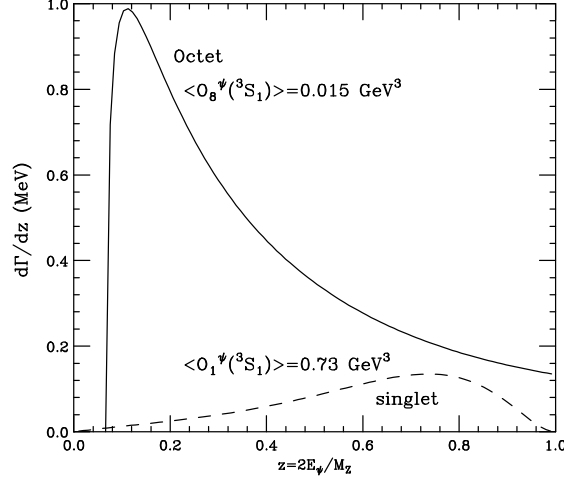


Fig. 2. Energy spectrum $d\Gamma/dz$ of ψ from the color-octet process $Z \rightarrow \psi q \bar{q}$ and the color-singlet process $Z \rightarrow \psi c \bar{c}$.

$L/(L+T) \approx 31\%$, while the dominant color-octet process gives a ratio $L/(L+T) \approx 20\%$. Nevertheless, the small production rate of prompt ψ renders this test rather difficult.

Concluding this section the color-octet process $Z \rightarrow \psi q \bar{q}$ contributes dominantly to the prompt ψ production, and the resulting branching ratio is consistent with the data. In addition, the energy spectrum of ψ serves as a useful test to distinguish between the color-octet process $Z \rightarrow \psi q \bar{q}$ and the color-singlet process $Z \rightarrow \psi c \bar{c}$.

3. ψ Production in Υ Decay¹⁷

The available experimental data on charmonium production in Υ decay are listed as follows,

$$\text{Br}(\Upsilon \rightarrow \psi + X) \begin{cases} = (1.1 \pm 0.4) \times 10^{-3} & \text{CLEO}^{24}, \\ < 1.7 \times 10^{-3} & \text{Crystal Ball}^{25}, \\ < 0.68 \times 10^{-3} & \text{ARGUS}^{26}. \end{cases} \quad (8)$$

Such large branching ratios by CLEO can hardly be explained by the CSM because the lowest order color-singlet processes are of order α_s^6 and there is no quantitative calculation of such existing in the literature. Only until a few years ago an indirect ψ production via the intermediate physical χ_c states was performed by Trottier²⁷. This process is of order α_s^5 , i.e., $1/\alpha_s$ larger than the color-singlet processes. Nevertheless, the branching ratio obtained is still one order of magnitude below the data. Therefore, it would be interesting to identify important color-octet processes.

The NRQCD factorization formula¹⁰ for the inclusive charmonium production in bottomonium decay, such as $\Upsilon \rightarrow \psi + X$, is given by

$$d\Gamma(\Upsilon \rightarrow \psi + X) = \sum_{m,n} d\hat{\Gamma}_{mn} \langle \Upsilon | O_m | \Upsilon \rangle \langle O_n^\psi \rangle, \quad (9)$$

where $d\hat{\Gamma}_{mn}$ are the short-distance factors for a $b\bar{b}$ pair in the state m to decay into a $c\bar{c}$ pair in the state n plus anything, where m, n denote collectively the color, total spin, and orbital angular-momentum of the heavy quark pair. $d\hat{\Gamma}_{mn}$ can be calculated in perturbation theory as a series expansion in $\alpha_s(m_c)$ and/or $\alpha_s(m_b)$. The nonperturbative factor $\langle \Upsilon | O_m | \Upsilon \rangle$ is proportional to the probability for the $b\bar{b}$ pair to be in the state m inside the physical bound state Υ , while $\langle O_n^\psi \rangle$ is proportional to the probability for a point-like $c\bar{c}$ pair in the state n to form the bound state ψ . The relative importance of the various terms in the above double factorization formula (9) can be determined by the order of v_b or v_c in the NRQCD matrix elements and the order of α_s in the short-distance factors $d\hat{\Gamma}_{mn}$. In fact, the calculation by Trottier²⁷ is an example of this factorization formula. An infrared divergence occurs in the leading order calculation of the short-distance factor, therefore, in addition to the color-singlet matrix element $\langle O_1^{\chi_{cJ}}(^3P_J) \rangle$, one also needs to include the color-octet matrix element $\langle O_8^{\chi_{cJ}}(^3S_1) \rangle$ to absorb the infrared divergence. In this case, the introduction of the color-octet matrix element is required by perturbative consistency.

For color-octet processes we first consider the produced $c\bar{c}$ pair in the color-octet 1S_0 , 3S_1 , or 3P_J configuration, which subsequently evolves into physical ψ described by the matrix elements $\langle O_8^\psi(^1S_0) \rangle$, $\langle O_8^\psi(^3S_1) \rangle$, or $\langle O_8^\psi(^3P_J) \rangle$, respectively. The matrix element $\langle O_8^\psi(^3S_1) \rangle$ has been extracted from the CDF data^{9,11,12}, while two different combinations of the other color-octet matrix elements have been extracted from the CDF data¹² and from the photoproduction data by Amundson *et al*¹³. Though of much smaller effects, we also consider the contributions by the higher Fock state of the color-octet $b\bar{b}$ pair inside the Υ associated with the matrix element $\langle \Upsilon | O_8(^3S_1) | \Upsilon \rangle$, whose value has not yet been determined. An order of magnitude of this matrix element can, in principle, be estimated by considering the ratio

$$\frac{\langle \Upsilon | O_8(^3S_1) | \Upsilon \rangle}{\langle \Upsilon | O_1(^3S_1) | \Upsilon \rangle} \frac{\langle O_1^\psi(^3S_1) \rangle}{\langle O_8^\psi(^3S_1) \rangle} \sim \left(\frac{v_b^2}{v_c^2} \right)^2, \quad (10)$$

which implies $\langle \Upsilon | O_8(^3S_1) | \Upsilon \rangle \approx 3 \times 10^{-3} \text{ GeV}^3$. The ratio in (10) tells us that processes associated with a color-octet $c\bar{c}$ pair inside the produced ψ and a color-singlet $b\bar{b}$ pair inside the decaying Υ are much more important than those with a color-octet $b\bar{b}$ pair inside the Υ and a color-singlet $c\bar{c}$ pair inside the ψ . However, such a large value for this matrix element would substantially increase the hadronic width of Υ , which would diminish the leptonic branching ratio to an unacceptable level. In order not to spoil the experimental value for the leptonic branching ratio and the total hadronic width of Υ , we necessarily obtain the following bound on $\langle \Upsilon | O_8(^3S_1) | \Upsilon \rangle$:

$$\langle \Upsilon | O_8(^3S_1) | \Upsilon \rangle \approx \left(1.9 \begin{array}{c} +5.1 \\ -4.6 \end{array} \right) \times 10^{-4} \text{ GeV}^3. \quad (11)$$

Therefore, the color-octet processes associated with $\langle \Upsilon | O_8(^3S_1) | \Upsilon \rangle$ have very small branching ratios, and thus negligible.

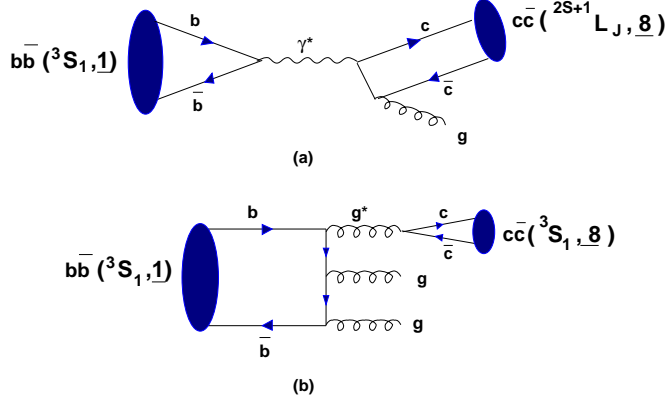


Fig. 3. Some of the contributing Feynman diagrams for the short-distance processes: (a) $b\bar{b}(\bar{}^3S_1, \underline{1}) \rightarrow \gamma^* \rightarrow c\bar{c}(\bar{}^{2S+1}L_J, \underline{8}) + g$; (b) $b\bar{b}(\bar{}^3S_1, \underline{1}) \rightarrow g^* gg \rightarrow c\bar{c}(\bar{}^3S_1, \underline{8}) + gg$;

The first color-octet process we consider is the one with the short distance process $b\bar{b}(\bar{}^3S_1, \underline{1}) \rightarrow c\bar{c}(\bar{}^{2S+1}L_J, \underline{8}) + g$, which is of order $\alpha^2\alpha_s$. One of the Feynman diagrams is shown in Fig. 3(a). Using the heavy quark spin symmetry relation $\langle O_8^\psi(\bar{}^3P_J) \rangle \approx (2J+1)\langle O_8^\psi(\bar{}^3P_0) \rangle$ ¹⁰, the total width from these processes can be simplified as

$$\begin{aligned} \Gamma_a(\Upsilon \rightarrow \psi + X) &= \frac{4\pi^2 Q_c^2 Q_b^2 \alpha^2 \alpha_s}{3} \frac{\langle \Upsilon | O_1(\bar{}^3S_1) | \Upsilon \rangle}{m_b^4 m_c} \left\{ \langle O_8^\psi(\bar{}^1S_0) \rangle (1 - \xi) \right. \\ &\quad \left. + \frac{\langle O_8^\psi(\bar{}^3P_0) \rangle}{3m_c^2} \left[\frac{(1 - 3\xi)^2}{1 - \xi} + \frac{6(1 + \xi)}{1 - \xi} + \frac{2(1 + 3\xi + 6\xi^2)}{1 - \xi} \right] \right\}. \end{aligned} \quad (12)$$

Using¹² $\langle O_8^\psi(\bar{}^1S_0) \rangle \approx \langle O_8^\psi(\bar{}^3P_0) \rangle / m_c^2 \approx 10^{-2} \text{ GeV}^3$, the contribution from the above color-octet processes to the inclusive branching ratio $\text{BR}(\Upsilon \rightarrow \psi + X)$ is only 1.6×10^{-5} , which is almost two orders of magnitude below the CLEO data (8).

The next process we consider is $b\bar{b}(\bar{}^3S_1, \underline{1}) \rightarrow ggg^* \rightarrow c\bar{c}(\bar{}^3S_1, \underline{8}) + gg$, which is of order α_s^4 . Fig. 3(b) shows one of the six Feynman diagrams for the $b\bar{b}(\bar{}^3S_1, \underline{1})$ pair annihilating into three gluons with one of the gluons converting into the $c\bar{c}(\bar{}^3S_1, \underline{8})$ pair. The partial width of this process is given by¹⁷

$$\Gamma_b(\Upsilon \rightarrow \psi + X) = \frac{5\pi\alpha_s^4}{486m_c^3 m_b^2} \langle \Upsilon | O_1(\bar{}^3S_1) | \Upsilon \rangle \langle O_8^\psi(\bar{}^3S_1) \rangle \times (0.571). \quad (13)$$

We note that the color-octet piece for the process $\Upsilon \rightarrow \chi_{cJ} + X$ considered by Trottier²⁷ can be obtained from the above Eq. (13) by simply replacing the matrix element $\langle O_8^\psi(\bar{}^3S_1) \rangle$ with $\langle O_8^{\chi_{cJ}}(\bar{}^3S_1) \rangle$. The prediction of the partial width is sensitive to the values of the two NRQCD matrix elements, the running coupling constant α_s , and the heavy quark masses. For convenience we can normalize this partial width to the three-gluon width $\Gamma(\Upsilon \rightarrow ggg)$

$$\frac{\Gamma_b(\Upsilon \rightarrow \psi + X)}{\Gamma(\Upsilon \rightarrow ggg)} = \frac{\pi\alpha_s}{8} \frac{\langle O_8^\psi(\bar{}^3S_1) \rangle}{m_c^3} \frac{0.571}{\pi^2 - 9}. \quad (14)$$

With $\langle O_8^\psi(^3S_1) \rangle = 0.014 \text{ GeV}^{39,12}$, and assuming $\text{BR}(\Upsilon \rightarrow ggg) \approx \text{BR}(\Upsilon \rightarrow \text{light hadrons}) = 0.92^{28}$, we obtain

$$\begin{aligned} \frac{\Gamma_b(\Upsilon \rightarrow \psi + X)}{\Gamma_{\text{total}}(\Upsilon)} &= \frac{\Gamma_b(\Upsilon \rightarrow \psi + X)}{\Gamma(\Upsilon \rightarrow ggg)} \times \text{BR}(\Upsilon \rightarrow ggg) \\ &\approx 2.5 \times 10^{-4}. \end{aligned} \quad (15)$$

This prediction is smaller than the CLEO data by merely a factor of 4, and is consistent with the bounds from Crystal Ball and ARGUS.

The above color-octet process can be extended to ψ' as well, simply by replacing the matrix element $\langle O_8^\psi(^3S_1) \rangle$ with $\langle O_8^{\psi'}(^3S_1) \rangle$, whose value has also been extracted from the CDF data to be $0.0042 \text{ GeV}^{39,11,12}$. With²⁸ $\text{BR}(\psi' \rightarrow \psi + X) \approx 57\%$, we obtain a branching ratio of 4.3×10^{-5} for the inclusive ψ production in the Υ decay that comes indirectly from ψ' . One can also consider the processes $\Upsilon \rightarrow \gamma g g^*$ followed by $g^* \rightarrow \psi$ (ψ') via the color-octet mechanism, and $\Upsilon \rightarrow g g \gamma^*$ followed by $\gamma^* \rightarrow \psi$ (ψ') in the color-singlet model. Their partial widths are suppressed by factors of $8\alpha/(15\alpha_s) \sim 0.02$ and $32\alpha^2 \langle O_1^\psi(^3S_1) \rangle / (45\alpha_s^2 \langle O_8^\psi(^3S_1) \rangle) \sim 0.06$, respectively, compared with the width of Eq.(13). Thus they contribute a branching fraction about 2×10^{-5} in the inclusive decay $\Upsilon \rightarrow \psi + X$. The indirect contribution from the ψ' from these two processes is about 6×10^{-6} .

Another process we consider is that the $b\bar{b}$ pair annihilates into a $q\bar{q}$ pair via the s -channel photon γ^* , and then a bremsstrahlung virtual gluon emitted from the light quark line splits into an octet $c\bar{c}(^3S_1, \underline{8})$, which then turns into ψ . This process is of order $\alpha^2\alpha_s^2$, which is similar to the dominant color-octet process in the prompt ψ production in Z^0 decay¹⁵. The partial width is given by

$$\begin{aligned} \frac{\Gamma_{1e}(\Upsilon \rightarrow \gamma^* \rightarrow q\bar{q}\psi)}{\Gamma(\Upsilon \rightarrow \gamma^* \rightarrow q\bar{q})} &= \frac{\alpha_s^2(2m_c)}{36} \frac{\langle O_8^\psi(^3S_1) \rangle}{m_c^3} \left\{ 5(1 - \xi^2) - 2\xi \log \xi \right. \\ &+ \left[2 \text{Li}_2\left(\frac{\xi}{1+\xi}\right) - 2 \text{Li}_2\left(\frac{1}{1+\xi}\right) - 2 \log(1+\xi) \log \xi \right. \\ &\left. \left. + 3 \log \xi + \log^2 \xi \right] (1+\xi)^2 \right\}, \end{aligned} \quad (16)$$

with $\xi = (m_c/m_b)^2$. Numerically, the ratio in (16) is only 8.3×10^{-6} , and so this process can be safely ignored.

Among all contributions, the largest comes from the process shown in Fig. 3(b), which has a branching ratio about 2.5×10^{-4} . The next largest contribution is the indirect process considered by Trottier²⁷, which gives a branching ratio of 5.7×10^{-5} . Other comparable processes are (1) the process shown in Fig. 3(a) which has a branching ratio of 1.6×10^{-5} , (2) the processes $\Upsilon \rightarrow g g \gamma^* \rightarrow \psi + X$ and $\Upsilon \rightarrow \gamma g g^* \rightarrow \psi + X$ via the color-octet mechanism which have a combined branching ratio about 2×10^{-5} , and finally, (3) the indirect contribution from ψ' having a branching ratio about 5×10^{-5} . Therefore, adding up the contributions from all these processes, we obtain a branching ratio $\text{BR}(\Upsilon \rightarrow \psi + X) \approx 4 \times 10^{-4}$, which is within 2σ of the CLEO data.

4. Conclusions

The color-singlet model, which has been popularly used for more than 15 years, is inadequate to describe the present data at the Tevatron, LEP, Υ decay, and fixed target experiments. The NRQCD description of quarkonium production provides a consistent theoretical framework and implies the so-called color-octet mechanism. It can explain the large p_T ψ and ψ' data at the Tevatron by fitting a small set of parameters of matrix elements. If the color-octet mechanism is correct, it would have a strong impact on other production modes. In this talk, we summarized the ψ production in Z^0 and Υ decays. The color-octet mechanism turns out to be very important in these modes. The color-octet process $Z \rightarrow \psi q \bar{q}$ can explain the data on ψ production in Z^0 decay. A more decisive test would be a measurement of the energy profile of the ψ where a very soft spectrum is predicted by the color-octet mechanism. Furthermore, in Υ decay the color-octet mechanism makes possible a new short distance process $b(\bar{3}S_1, \underline{1}) \rightarrow g g g^* \rightarrow g g c \bar{c}(\bar{3}S_1, \underline{8})$, which can bring the theoretical prediction to be within 2σ of the central value of the CLEO data.

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